Ruled Surfaces for Rationalization and Design in Architecture

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Abstract

In this work we address the challenges in the realization of free-form architecture and complex shapes in general with the technical advantages of ruled surfaces. We propose a geometry processing framework to approximate (rationalize) a given shape by one or multiple strips of ruled surfaces. We discuss techniques to achieve an overall smooth surface and develop a parametric model for the generation of curvature continuous surfaces composed of ruled surface strips. We illustrate the usability of the proposed process at hand of several projects, where the pipeline has been applied to compute NC data for mould production and to rationalize large parts of free-form facades.

1 Introduction

The complexity of contemporary free-form architecture has been a driving force for the development of new digital design processes over the last years. Mainstream design tools and CAD software technology often adapt decade old methods from the automotive and other industries leaving a certain realization gap challenging engineers. In recent years, several attempts have aimed at filling this gap by providing architecture with tailor-made computational tools for design and production. Applied mathematics and in particular geometry have initiated the implementation of comprehensive frameworks for modeling and mastering the complexity of today’s architectural needs (Eigensatz et al 2010; Fu et al. 2010; Pottmann et al. 2010; Singh and Schaefer 2010).

From a geometric point of view, a shape's complexity may be judged at hand of a curvature analysis. A typical approach classifies surface patches as *planar*, single curved (=*developable*, as in the work of F. Gehry) or general *double curved* free-form (Pottmann et al. 2007). An interesting class of double curved surfaces, so called *ruled surfaces* (cf. Fig. 1), comprises developable surfaces and share their property of being generated by a continuously moving straight line. However, only single curved surfaces may be unfolded to the plane without stretching or tearing. With general ruled surfaces giving up on this property, we gain an interesting amount of flexibility that we seek to take advantage from in this work. Apart from that, the one parameter family of straight lines on a ruled surface opens a wide range of advantageous options for support structures, mould production or facade elements, to name a few.

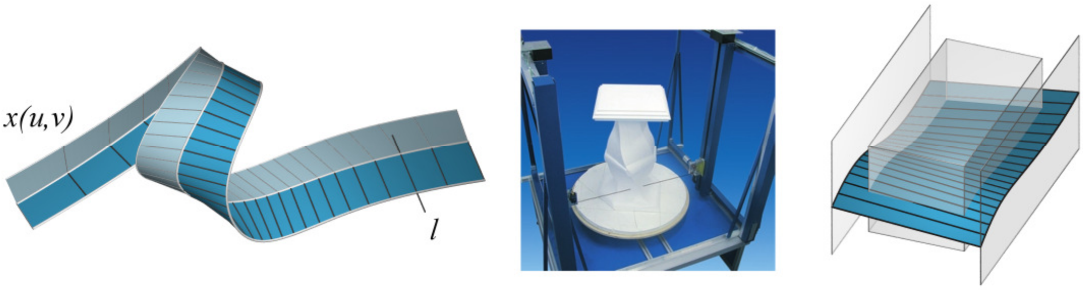

Figure 1. A ruled surface is formed by a continuous family of straight line segments *l*. A ruled surface strip model is composed of several ruled surface patches, glued together in a smooth way (left). Many production processes are based on ruled surfaces, e.g. heated wire cutting (center and right), where the heated wire moves on a ruled surface. Our goal is to approximate free-form shapes by ruled surfaces to take advantage of cost-effective fabrication options.

Ruled surfaces have been popular in architecture probably not only since the pioneering works of A. Gaudí and V. Shukhov. With their strong focus on structural elegance, these and many other contributions are in contrast to recent free-form architecture. In the following, we propose an automated framework to describe (*rationalize*) free-form shapes in an optimal sense by ruled surfaces. We will do so at a large scale to approximate entire facades by single patches of ruled surfaces and - if this does not suffice - by multiple strips of ruled surfaces glued together. At a smaller scale, we will show how to provide NC data for the cost-effective production of free-form moulds, for example on heated wire cutting machines. All these applications are embedded into a new digital design process (cf. Fig. 2). Following the presented work-flow, the needs of customers have been addressed successfully in several recent projects, of which some will be presented below.

A sketchy rationalization of free-form shapes with ruled surfaces may be achieved with existing CAD software, which do not support smooth ruled surface strip models. Compared to techniques such as interpolation or approximation of rectangular point grids or connecting two input curves by straight line segments through rail sweeps or lofts, the herein proposed framework significantly improves both on automation level and accuracy (minimal rationalization error). We begin our discussion by recalling how ruled surfaces are conveniently described with off-the-shelf CAD software. Then, we explore the approximation of a given shape by one or multiple patches of ruled surfaces. We show how multiple strips of ruled surfaces may be joined with varying degree of smoothness and present a parametric model for ruled surface strip models. In particular, our contribution comprises

- an efficient method for approximating a given shape by a ruled surface, that naturally generalizes to strips of ruled surfaces;
- an enhancement to this basic approximation algorithm that joins strips of ruled

surfaces with approximate tangent continuity;

- a design tool for curvature continuous ruled strip models;
- the application of these algorithms to the rationalization of entire free-form facades and to the computation of NC data for the economical production of free-form moulds.

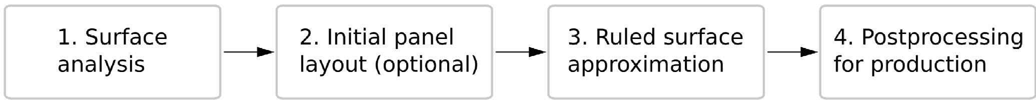

Figure 2. Overview of the proposed workflow for the employment of ruled surfaces in free-form architecture.

2 Ruled Surface Approximation

For the theoretical foundation of the different stages of the workflow in Fig. 2, we require some knowledge about the geometry of ruled surfaces that we summarize below. On top of this theory, we will formulate the basic ruled surface approximation algorithm that will take us to the first application example.

2.1 Geometry of Ruled Surfaces

B-spline curves (and NURBS curves, their more general superset) are versatile tools that have found their way into nearly any CAD software package. The simplest B-spline curve is that of polynomial degree 1 – the straight connection line of two input control points. Let us take two B-spline curves, denoted by a and b, which are of the same degree n and parameterized over the same interval. If we join all pairs of points $a(u)$ and $b(u)$ to the same parameter value u by a straight line segment (a degree 1 B-spline curve), we obtain a ruled surface strip connecting the two input curves. This is a special case of the well known tensor product B-spline surface construction (Pottmann et al. 2007). Adding more input curves, that are one after another connected by straight lines, we obtain a ruled surface strip model. Its control points are convenient handles to modify the shape. We will make use of this below. As a technical detail, we want to assume that the B-spline surface's knot vectors are fixed and uniform.

Ruled surfaces have many interesting geometric properties. Any surface point p is element of a straight line (a *ruling*) laying entirely on the ruled surface. Clearly, the surface does not curve if we move from p in ruling direction. Such a tangential direction is said to be of vanishing normal curvature and called an *asymptotic direction* (Fig. 4, left). Excluding developable surfaces from our discussion, ruled surfaces are known to be of *negative Gaussian curvature* (locally saddle shaped) in all their surface points. Hence, they exhibit another tangential direction of zero normal curvature different from the ruling direction at any point (do Carmo 1976).

This short excursion into the geometry of ruled surfaces motivates the first two steps of the process workflow depicted in Fig. 2. For the ease of discussion, let us assume that the input shape is given as a dense point cloud. In the first step, we estimate the Gaussian curvature in any point of the model (Yang and Lee 1999) and discard regions with positive Gaussian curvature, where an approximation with a (double curved) ruled surface would not make much sense. As a result, we obtain a rough segmentation of the input shape into patches that will be subject to ruled surface approximation (Fig. 3, right).

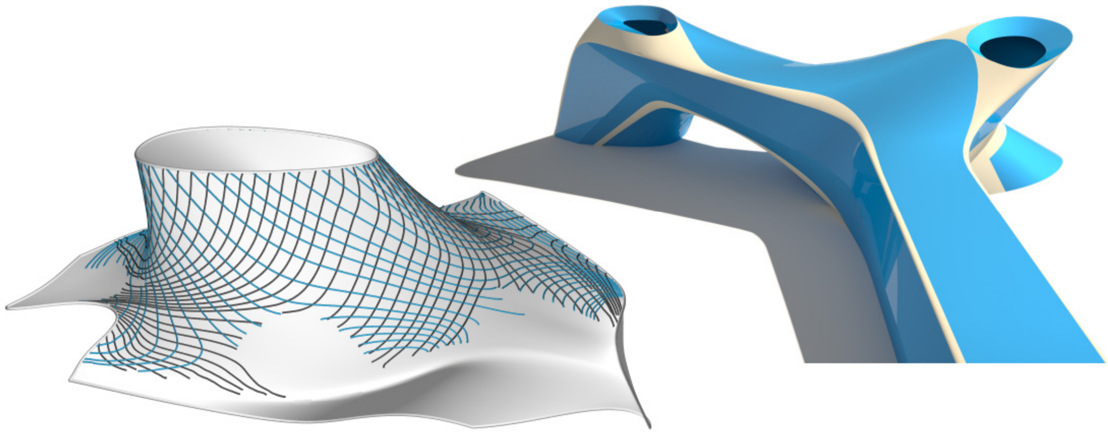

Figure 3. Curvature analysis yields a segmentation of the input shape into patches feasible for ruled surface approximation (right). The asymptotic curves follow the directions of vanishing normal curvature on a surface. In negatively curved (i.e., locally saddle shaped) regions there are two asymptotic directions in each surface point. If one family of asymptotic curves in a region is not curved too much, this indicates a good possibility for approximating that region by a ruled surface (left).

In the second step, we estimate the asymptotic directions on a sufficient number of points on each patch. Integration of the resulting tangent vector field yields curves which follow the asymptotic directions (see Fig. 3, left). These *asymptotic curves* give a hint at the ruling directions of an approximating ruled surface. If the given patch is supposed to be broken down further into production sized panels, this panel layout may be derived from the asymptotic curve network, if not provided by the customer's design team. In either case, we use the estimated asymptotic direction to automatically construct an initial ruled surface, naturally of non-optimal approximation quality, by aligning rulings with asymptotic curves. The next section (step 3 of the processing pipeline) will discuss how the shape of this initial approximation is modified to optimally fit the given target shape.

2.2 Basic Approximation Algorithm

Let us briefly summarize the input to this stage of the pipeline. The target shape is given as a dense point cloud *P*, describing the intended shape. Moreover, we have already set up an initial ruled B-spline surface $x_0(u,v)$, roughly approximating *P*. The goal of this third stage is to compute an improved version x of x_0 that optimally approximates P (cf. Fig. 4, right). In mathematical terms we may say that a surface x approximates P optimally, if the squared distance $d^2(x,P)$ of P to x is minimal. This motivates the computation of the final ruled surface approximation x as result of the minimization problem (Pottmann and Leopoldseder 2003),

$$\min d^2(x,P) + w \cdot f_{smooth}(x).$$

The unknowns of this non-linear optimization problem are the control points of the approximating surface x. We solve the problem numerically with a Gauss-Newton method (Nocedal and Wright 1999). The second term $f_{smooth}(x)$ added to the distance term is a smoothing term ensuring both numerical stability of the minimization and a visually pleasing final solution. The weight w controls the influence of the smoothing term. The larger w, the smoother the final solution will be, by giving up on approximation quality at the same time. Please note that above optimization works for single patches as well as for ruled surface strip models. Previous work on ruled surface approximation draws its motivation from production technologies, in particular cylindrical flank milling (Stute et al. 1979; Senatore et al. 2008; Sprott and Ravani 2008), and from reconstruction problems in computer aided geometric design (Hoschek and Schwanecke 1998; Chen and Pottmann 1999). Our approach extends the work of (Pottmann and Leopoldseder 2003), who generalize the concept of active contours from image processing to active shape models for surface fitting.

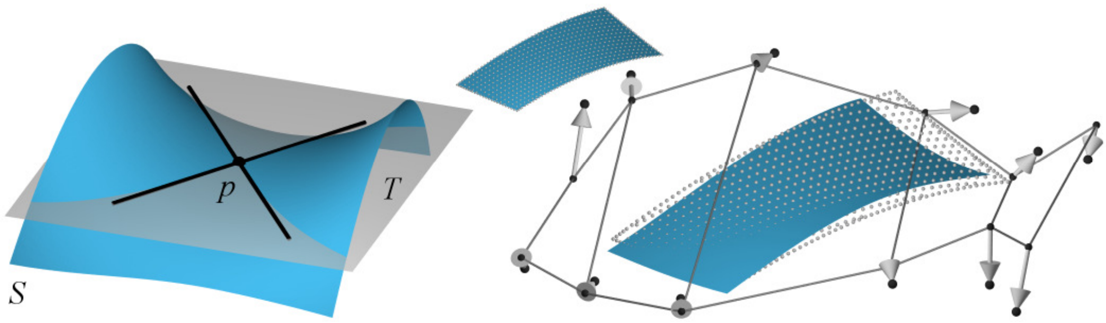

Figure 4. (Left) In a point *p* of negative Gaussian curvature (locally saddle shaped), intersection of a surface *S* with its tangent plane *T* yields an intersection curve with a double point in *p*. The tangents to the intersection curve in *p* are the asymptotic directions of *S* in *p* (depicted in black). (Right) The control points of a ruled B-spline surface are displaced to optimally approximate a given point cloud. The displacement vectors for the final optimal approximation (center, top) are obtained as solution of a mathematical optimization problem.

3 Heated Wire Cutting for Mould Production

The basic approximation algorithm outlined above describes the third step in the digital design workflow for ruled surface approximation. In order to complete the outline, we are going to discuss the entire process chain at hand of a geometry processing for a facade detail of Beijing's SOHO Galaxy by Zaha Hadid Architects (cf. Fig. 6). The given surface was to be realized by glass-fibre reinforced concrete panels, using in part heated wire cut EPS foam moulds for free-form panel production. In an abstract setting, the heated wire moves on a ruled surface through the EPS block (Fig. 1, right).

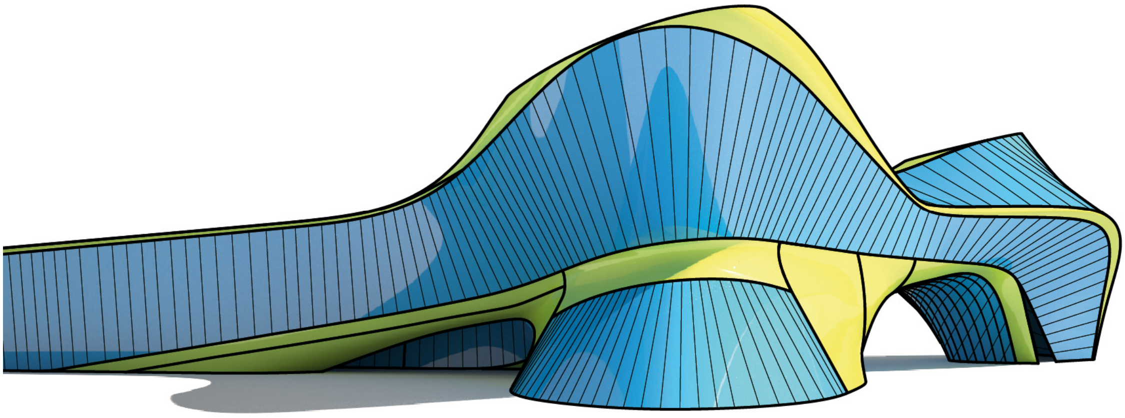

Figure 5. Large parts of the facade of the Cagliari Contemporary Arts Center by Zaha Hadid Architects have been rationalized with ruled surfaces.

A Gaussian curvature analysis of the surface indicates that the entire facade cutout is feasible for ruled surface approximation. The panel layout has been guided by the asymptotic and principal curvature lines and was constrained by the maximum panel production size of 2x1.4 meters. The final 81 surface patches have been approximated by ruled surfaces with a maximal approximation error of 2.4 millimeters (see Fig. 6). The so obtained ruled surface patches are transformed into a local EPS block coordinate system, where intersection with the portal planes yields the geometric description of the NC data. Conversion of this data onto the machine by keeping the correct correspondences of points on the intersection curves completes the geometry processing.

Table 1: Geometric continuity in a surface point *p*.

Order of Geometric Continuity	The two strips of ruled surfaces …
zero-order	… share *p* as common point.
first-order (tangent continuity)	… share the same unit surface normal in *p*.
second-order (curvature continuity)	… exhibit coinciding osculating paraboloids (identical curvature behavior) in *p*.

4 Tangent Continuous Strips of Ruled Surfaces

Above example employs single patches of ruled surfaces only. If we join multiple ruled surfaces to a strip model, we require a certain degree of smoothness across common strip boundaries to achieve satisfactory visual appearance. The most popular mathematical tool for smoothness is the concept of parametric C^k continuity, requiring coinciding derivatives of order k in common boundary points. However, this is too restrictive for our and many other CAD purposes and the concept of geometric continuity (Peters 2002), that we briefly summarize in Table 1, provides a suitable alternative.

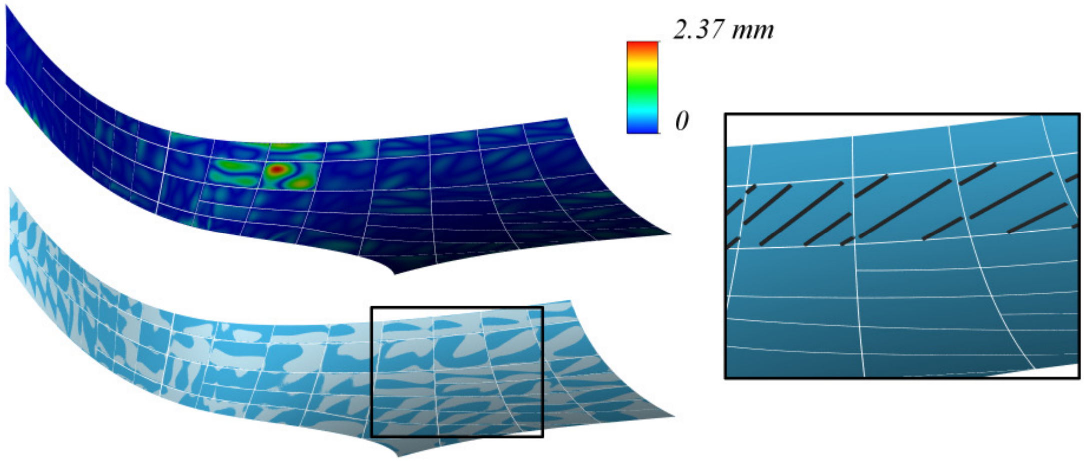

Figure 6. (Top, left) For a 54 m² facade detail of SOHO Galaxy by Zaha Hadid Architects, the maximal deviation of the ruled surface approximation is below 2.4 millimeters. (Bottom, left) shows both reference (grey) and approximated ruled panels (blue). The zoom-out (right) illustrates the ruling direction for some panels.

In the following, we will extend the basic approximation algorithm to optimize for tangent continuous ruled surface strip models. Consider t, the common boundary curve tangent of two strips meeting in a point p (cf. Fig. 8). Obviously, t is tangent to both strips. Moreover, let r denote the ruling direction of one strip and s the ruling direction of the other strip. Then, r and t span the tangent plane of the one strip and t and s the tangent plane of the other strip. If the two strips meet tangent continuous in p, the two tangent planes coincide. If not, the volume of the parallelepiped defined by r, s and t will be non-vanishing.

This last observation motivates an additional penalty function for the basic approximation algorithm. We sample common boundary curves of joining strips in a user-defined number and add for each sample p_k the squared volume of the parallelepiped Q_k spanned by the unit ruling directions r_k and s_k and the common normalized boundary tangent direction t_k,

$$\min d^2(x,P) + w_0 \cdot f_{smooth}(x) + w_1 \cdot \sum_k \det^2(r_k, s_k, t_k)\ .$$

Please note that tangent continuity not only requires matching tangent planes but also that the surface normals point in the same direction. In our approach, the regularization term will prevent from flipping surface normals.

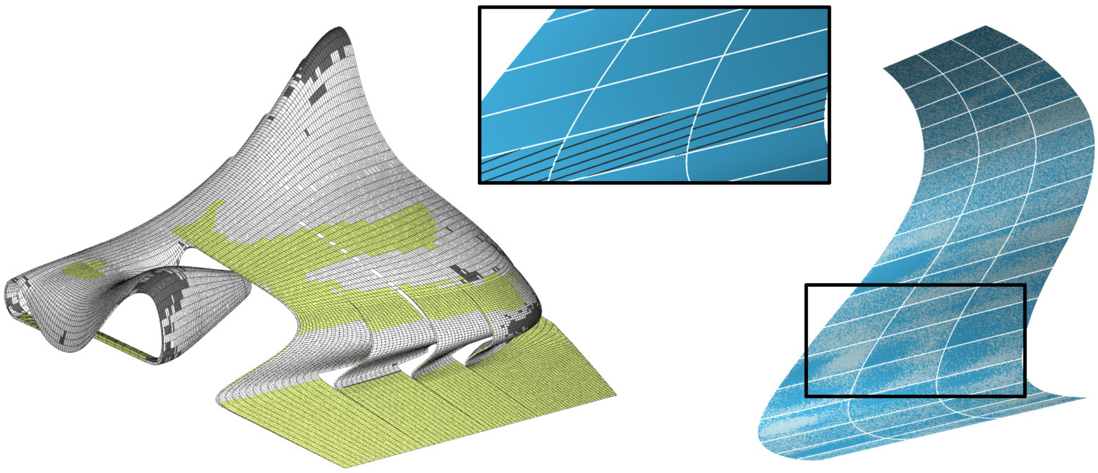

Figure 7. The panels of the Heydar Aliyev Cultural Centre facade by Zaha Hadid Architects are classified for planar (green), ruled (light grey) or general double curved (dark grey) rationalization (left). The ruled surface approximations of a subset of 51 panels are shown on the right, with some ruling directions visualized in a zoom-out (center).

5 Cagliari Contemporary Arts Center

Let us illustrate the additional penalty for tangent continuity at hand of the geometry processing of the facade of the Cagliari Contemporary Arts Center by Zaha Hadid Architects. The facade part exhibits a total of 13.500 m^2 of which 85% are found to be feasible for ruled surface approximation in the surface analysis step. The facade's shape suggests a natural segmentation into maximally sized patches (see Fig. 3, right) that are subsequently approximated by a single ruled surface patch each (except the saddle shaped detail, see below). Apart from the roof, for which a deviation of 400 millimeters was allowed to achieve additional smoothness, the approximation quality is within millimeter tolerance to the original surface (see Fig. 5).

The geometry of the saddle shaped detail (cf. Fig. 9) renders an approximation with a single ruled surface patch infeasible. For this reason, we decided in favor of an approximation with a ruled surface strip model including above module for tangent continuity. The boundary curves between adjacent strips were further constrained to lie in a family of planes meeting in a common intersection line.

6 Curvature Continuous Strips of Ruled Surfaces

By a well known result from differential geometry, the osculating paraboloid in a surface point is well defined by three tangential directions and the normal curvatures therein (do Carmo 1976). Let us assume that the strip model is tangent continuous in p. Then, the two strips share the boundary curve tangent t in p. As the unit surface normals in p

coincide, t exhibits the same normal curvature on either strip. For the remaining two directions that we require, recall that in any point of a general ruled surface there are two directions of vanishing normal curvature. If we assume that the two strips' asymptotic directions coincide in p, we obtain curvature continuity.

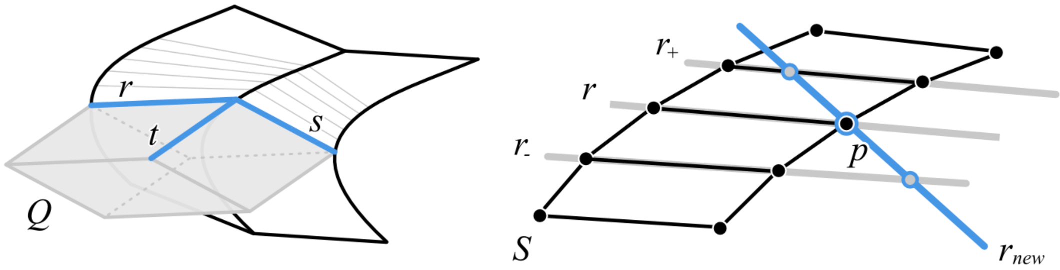

Figure 8. Tangent continuity of a ruled surface strip model implies vanishing volume of the parallelepiped *Q* spanned by the ruling directions *r* and *s* and the common boundary curve tangent *t* (left). The next strip's ruling in point *p* of a discrete curvature continuous strip model *S* is obtained as common intersection line r_{new} of r_-, *r* and r_+ (right).

We arrive at the following two cases. If the ruling direction of the first strip (which is an asymptotic direction) matches the ruling direction of the second strip, we simply obtain a larger ruled surface patch. For the second, more interesting case, the ruling direction of the first strip aligns with the general asymptotic direction of the second strip, and vice versa. We obtain a significant zigzag pattern of rulings (cf. Fig. 10, left).

We found the curvature continuity property being too restrictive to be added as penalty function to the basic approximation algorithm. However, we may setup a parametric model for discrete, curvature continuous, ruled strip models. Consider an initial strip of quadrilaterals S as in Fig. 8, right. In a vertex p, we compute the next strip's ruling as common intersection line of the first strip's ruling in p and those of its adjacent boundary points. In a refinement process, the set of intersection lines converges towards the so called *Lie quadric* which is in second-order contact with either ruled surface strip along the entire ruling (Pottmann and Wallner 2001). The shape of the first ruled strip determines the shape of the entire parametric model and may be modified by the user, along with the width of the single strips (Fig. 10).

7 Conclusions

The advantages of ruled surfaces have been known to architecture for centuries. In the present work, we outline a geometric processing pipeline to link ruled surfaces to contemporary free-form architecture. We show how a given complex shape is segmented into regions subsequently approximated by a single patch or strips of ruled surfaces. We discuss enhancements to this basic approximation algorithm to increase the smoothness across common strip boundaries and introduce a parametric model for curvature continuous ruled surface strip models. The discussed process has been applied successfully to compute optimal NC data for mould production and perform large scale facade approximation.

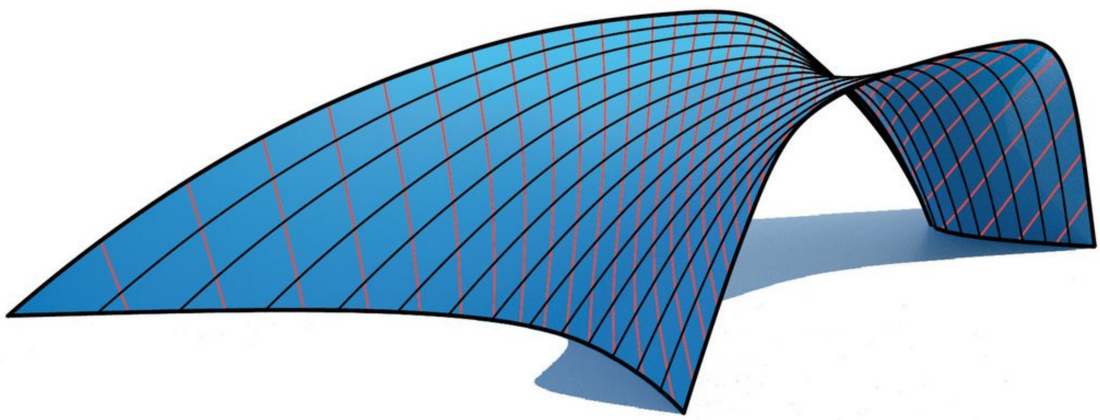

Figure 9. This saddle surface, a detail from the facade of the Cagliari Contemporary Arts Center, has been rationalized with 14 ruled surface strips. The strips have been simultaneously optimized towards a smooth overall surface under the constraint that the transition curves lie in user-specified vertical and coaxial planes.

8 Acknowledgements

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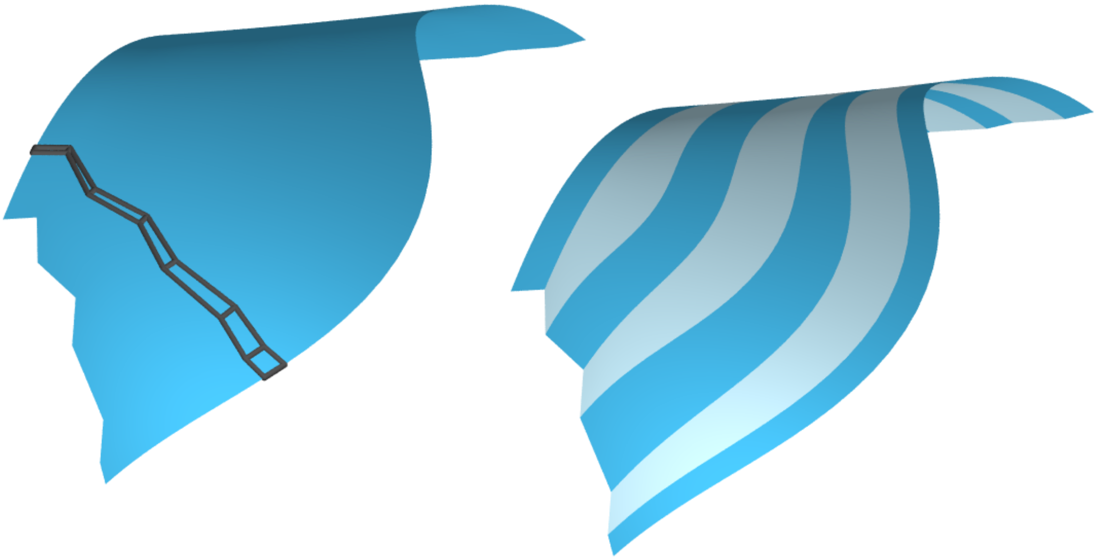

Figure 10. A curvature continuous surface formed by a sequence of ruled surface strips has been generated in a parametric model which starts with a single strip and automatically adds further strips so that curvature continuity is achieved. The shape is controlled by the first strip and the width function for the other strips. The left-hand side shows the zigzag pattern of rulings, which is necessary to obtain curvature continuity.

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